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A Closed-Form Approximation of Correlated Multiuser MIMO Ergodic Capacity with Antenna Selection and Imperfect Channel Estimation

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Abstract—Antenna selection (AS) is a promising technology to substantially reduce the complexity of massive multiple-input multiple-output (MIMO) systems. However, spatial correlation and imperfect estimation of channel state information (CSI) are well known to have a direct impact on the capacity of feasible MIMO schemes. In this paper, a tight closed-form approximation of the ergodic capacity for correlated Rayleigh fading multiuser MIMO channels with receive AS and imperfect CSI is presented. The derived expression takes into account the spatial correlation at both link sides and channel estimation error at the receiver. It can be used for arbitrary numbers of users, antennas, and receive RF chains. Furthermore, a concise analytical capacity formula is derived in the high signal-to-noise ratio (SNR) region. Numerical results validate the accuracy of our closed-form expressions over different channel conditions and SNRs. The new capacity approximation extends the state-of-the-art and enables efficient performance evaluation of varied multiantenna applications including massive MIMO for 5G systems.

Index Terms—Multiuser MIMO, antenna selection, ergodic capacity, spatial correlation, imperfect channel estimation.

I. INTRODUCTION

Wireless multiple-input multiple-output (MIMO) schemes have been adopted in the Long Term Evolution (LTE) standards of 4G systems as a key technology to meet the increasing demands for high data rate applications [1]. At present, Releases 13 and 14 of LTE-Advanced Pro (4.5G) support up to 16 and 32 antennas at the base station (BS), respectively with an increased number towards 8 co-scheduled users. In addition, the emerging massive MIMO with tens to hundreds of antennas at BS is expected to be an essential component in future 5G systems [1], [2]. To reduce the high complexity and consumed power burdens of MIMO schemes without significant performance loss, particularly for a large number of BS antennas and associated radio frequency (RF) chains, antenna selection (AS) has been considered as an

effective solution in LTE specifications [3]. This technique has been studied intensively to exploit the additional degrees of freedom (DoF) represented by the difference between higher number of multiple antennas and available RF chains [4]–[9]. Moreover, it has been practically investigated for the massive MIMO systems in real propagation channels and shown to achieve significant capacity gain and substantial improvement in the overall energy efficiency [10], [11].

Theoretically, the capacity of MIMO channels increases linearly with the minimum number of transmit and receive antennas assuming independent Rayleigh fading environment and fixed bandwidth and power conditions [12]. However, spatial correlation owing to insufficient antenna spacing and/or poor scattering can severely decrease the capacity of feasible systems [4], [13]–[15]. Besides, similar problem is typically induced due to an imperfect estimation of channel state information (CSI) when large number of MIMO channel parameters is required [16]. The CSI is commonly obtained using training sequences and it plays an important role in the signal detection process [17]. Therefore, the issues of channel correlation and imperfect CSI are considered as critical design objectives for MIMO systems with AS technology [18]–[20].

Generally, development of modern MIMO schemes requires efficient channel capacity evaluation as a key performance measure. Therefore, numerous analytical expressions have been derived and verified by simulations assuming different channel conditions. For instance, a spatial correlation was considered in [12] and [13] with perfect CSI at the receiver (CSIR) whereas independent and identically distributed (i.i.d.) Rayleigh fading processes were assumed in [16] with different scenarios of CSI error. In [17], an i.i.d. fading channel is considered with estimation error at both link ends for the low signal-to-noise ratio (SNR) region. However, a closed-form analytical capacity representation is still challenging topic for general real-world MIMO channels with AS diversity. In the literature, capacity bounds have been presented for spatially correlated MIMO channels with AS and perfect CSIR, but constrained to the selection of only one receive antenna in [5] and based on the instantaneous channel gains in [4].

In this paper, we consider the ergodic capacity of correlated multiuser MIMO (MU-MIMO) Rayleigh fading channel with receive AS (RAS) and imperfect CSI. The main contributions of the paper are: 1) a simplified closed-form expression of the ergodic capacity is derived. It considers more realistic environment of spatial correlation at each transmitter and BS receiver with imperfect CSIR in contrast to the previous works

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with AS in [4], [5], [18] and without AS in [12], [13], [16], [17]. Besides, it does not rely on the instantaneous channel gains as in [4] and neither restricted to the selection of only one receive antenna as in [5]; 2) the presented expression can be seen as a generalized form that capture different special case scenarios of single user MIMO, uncorrelated channels, and/or perfect CSIR; 3) a concise analytical expression is derived for the ergodic capacity at the high SNR region; 4) in view of LTE targets towards massive MIMO, the numerical results confirm the tightness of the presented analytic formulas over different channel conditions, a wide range of SNR, and arbitrary numbers of users, antennas, and receive RF chains.

The rest of this paper is organized as follows. In Section II, the system model is described. Section III presents the ergodic capacity analysis. Numerical results are shown in Section IV. Finally, Section V concludes the paper.

Notations: Bold-face uppercase and lowercase letters denote matrices and vectors, respectively. $\mathcal{C}^{m \times n}$ denotes complex $m \times n$ matrix and the superscripts $[\cdot]^H$ stands for conjugate transposition. $\mathbb{E}[\cdot]$ is the expectation operator and \mathbf{I}_m is $m \times m$ identity matrix. $\text{diag}\{\cdot\}$ denotes a diagonal matrix. $\mathbf{A}^{1/2}$ stands for the Hermitian square root of \mathbf{A} while $\mathbf{A}^{(l)}$ denotes $l \times l$ principal submatrix constructed from 1^{st} to l^{th} rows and columns of \mathbf{A} with $(i, j)^{th}$ element given as $[\mathbf{A}]_{ij}$.

II. SYSTEM MODEL

Consider an uplink spatial multiplexing MU-MIMO system of K users communicating simultaneously with one common BS over correlated Rayleigh flat-fading channel. Each user has N_T antennas while the BS is equipped with M antennas and employs RAS to connect $M_s \leq M$ antennas with the accessible M_s RF chains. For total transmit power constraint \mathcal{P} and only CSIR, equal power allocation \mathcal{P}/N with $N = KN_T$ is typically used for each transmit antenna since the power adaptation to channel variations will not increase the ergodic capacity [16]. Moreover, training based estimation of CSI for large MIMO channels may incur capacity loss in practice. Therefore, we assume that the CSI and estimation error are given a priori for signal detection and RAS, and the impact of required overhead on the ergodic capacity is beyond the scope of this paper.

Based on the well-known Kronecker model [4], [12], [18], the spatially correlated channel of user k is represented as $\mathbf{H}_k = \mathbf{R}_r^{1/2} \mathbf{G}_k \mathbf{R}_t^{1/2} \in \mathcal{C}^{M \times N_T}$ whose entries $[\mathbf{H}_k]_{mn}$; $m = 1, \dots, M$; $n = 1, \dots, N_T$ describe the complex gains from n^{th} transmit to m^{th} receive antennas. $\mathbf{G}_k \in \mathcal{C}^{M \times N_T}$ is spatially white channel of user k whose elements are i.i.d. complex Gaussian random variables with zero-mean and unit-variance. $\mathbf{R}_r \in \mathcal{C}^{M \times M}$ and $\mathbf{R}_t \in \mathcal{C}^{N_T \times N_T}$ denote positive definite correlation matrices at the receiver and transmitter of user k , respectively. The structures of \mathbf{R}_r and \mathbf{R}_t are realized using the exponential correlation model [13], [18], which provides more realistic results for large number of MIMO channels. Thus, the overall channel $\mathbf{H} \in \mathcal{C}^{M \times N}$ can be given as

$$\mathbf{H} = [\mathbf{H}_1 \dots \mathbf{H}_K] = \mathbf{R}_r^{1/2} [\mathbf{G}_1 \dots \mathbf{G}_K] \mathbf{R}_t^{1/2} = \mathbf{R}_r^{1/2} \mathbf{G} \mathbf{R}_t^{1/2} \quad (1)$$

where $\mathbf{G} \in \mathcal{C}^{M \times N}$ is the overall spatially white channel matrix and $\mathbf{R}_t = \text{diag}\{\mathbf{R}_{t,1}, \dots, \mathbf{R}_{t,K}\} \in \mathcal{C}^{N \times N}$ is the overall transmit correlation matrix. Note that the Kronecker model may still

underestimate the capacity of massive MIMO channels due its potential deficiencies in the structure. However, it provides a tractable approach to the analysis of different correlated MIMO channels [4-6, 12-14] including massive MIMO [18]. So, it is adopted in this study while more practical models for special scenarios that consider the interdependency between scattering distributions at both link sides can be found in [12].

From [16], the considered small-scale fading process with imperfect CSIR can be written as

$$\mathbf{H} = \tilde{\mathbf{H}} + \mathbf{E} = \mathbf{R}_r^{1/2} (\tilde{\mathbf{G}} + \mathbf{E}) \mathbf{R}_t^{1/2} \quad (2)$$

where $(\tilde{\mathbf{G}} + \mathbf{E})$ represents \mathbf{G} , $\tilde{\mathbf{H}} = \mathbf{R}_r^{1/2} \tilde{\mathbf{G}} \mathbf{R}_t^{1/2} \in \mathcal{C}^{M \times N}$ is the estimate of \mathbf{H} , and $\mathbf{E} = \mathbf{R}_r^{1/2} \mathbf{E} \mathbf{R}_t^{1/2} \in \mathcal{C}^{M \times N}$ is the estimation error matrix. Both $\tilde{\mathbf{G}} \in \mathcal{C}^{M \times N}$ and $\mathbf{E} \in \mathcal{C}^{M \times N}$ are spatially white matrices having i.i.d. entries of zero-mean complex Gaussian with $\sigma_{\tilde{\mathbf{G}}}^2$ and $\sigma_{\mathbf{E}}^2 = 1 - \sigma_{\tilde{\mathbf{G}}}^2$ variances, respectively.

For each symbol interval, the received signal vector $\mathbf{r}_s \in \mathcal{C}^{M_s \times 1}$ associated with RAS is given by

$$\mathbf{r}_s = \sum_{k=1}^K \tilde{\mathbf{H}}_k \mathbf{v}_k + \mathbf{n}_s = \mathbf{H}_s \mathbf{v} + \mathbf{n}_s \quad (3)$$

where $\tilde{\mathbf{H}}_k \in \mathcal{C}^{M_s \times N_T}$ and $\mathbf{H}_s \in \mathcal{C}^{M_s \times N}$ are the channel matrices extracted from \mathbf{H}_k and \mathbf{H} , respectively, $\mathbf{v}_k \in \mathcal{C}^{N_T \times 1}$ is the signal vector of k^{th} user, $\mathbf{v} \in \mathcal{C}^{N \times 1}$ is the overall signal vector of zero-mean and covariance matrix $\mathbb{E}[\mathbf{v}\mathbf{v}^H] = (\mathcal{P}/N) \mathbf{I}_N$, and $\mathbf{n}_s \in \mathcal{C}^{M_s \times 1}$ is the selected vector of i.i.d. additive white Gaussian noise elements having zero-mean and variance σ_n^2 . Note that full-array antenna selection of highest DoFs is considered in this study in contrast to that of [11].

III. ERGODIC CAPACITY OF CORRELATED MU-MIMO CHANNEL WITH RAS AND IMPERFECT CSIR

Based on the capacity analysis of MIMO channels in [12], [13], and [16], the lower bound capacity of signal model (3) with fixed channel realization \mathbf{H}_s can be written in bit/s/Hz as

$$C \geq \log_2 \det \left(\mathbf{I}_L + \frac{\rho/N}{1 + \sigma_{\mathbf{E}}^2 \rho} \mathbf{W} \right) \quad (4)$$

where $\rho = \mathcal{P}/\sigma_n^2$ denotes the average SNR at each receive antenna and $\mathbf{W} \in \mathcal{C}^{L \times L}$ with channel rank $L = \min\{N, M_s\}$ is defined as

$$\mathbf{W} = \begin{cases} \mathbf{H}_s \mathbf{H}_s^H = \tilde{\mathbf{R}}_r \mathbf{G}_s \mathbf{R}_t \mathbf{G}_s^H, & M_s \leq N \\ \mathbf{H}_s^H \mathbf{H}_s = \mathbf{G}_s^H \tilde{\mathbf{R}}_r \mathbf{G}_s \mathbf{R}_t, & M_s > N \end{cases} \quad (5)$$

where the submatrix $\mathbf{G}_s \in \mathcal{C}^{M_s \times N}$ is extracted from \mathbf{G} and $\tilde{\mathbf{R}}_r \in \mathcal{C}^{M_s \times M_s}$ is the receive correlation matrix after RAS.

Therefore, the ergodic capacity \bar{C} over randomly varying channel realizations can be evaluated as

$$\bar{C} = \mathbb{E}[C] \geq \mathbb{E} \left[\log_2 \det \left(\mathbf{I}_L + \frac{\rho/N}{1 + \sigma_{\mathbf{E}}^2 \rho} \mathbf{W} \right) \right]. \quad (6)$$

Using Jensen's inequality and since the term $[\log_2 \det(\cdot)]$ is a concave function, (6) can be approximated as

$$\bar{C} \approx \log_2 \left[\mathbb{E} \left\{ \det \left(\mathbf{I}_L + \frac{\rho/N}{1 + \sigma_{\mathbf{E}}^2 \rho} \mathbf{W} \right) \right\} \right]. \quad (7)$$

It follows from [12, Theorem II.3] and [21] that the term under expectation, $\det(\mathbf{\Delta}) = \det(\mathbf{I}_L + (\rho/N)\mathbf{W}/(1 + \sigma_{\tilde{\mathbf{e}}}^2\rho))$, can be written in terms of all principal minor determinants of \mathbf{W} as

$$\det(\mathbf{\Delta}) = 1 + \sum_{u=1}^L \sum_{l=1}^u \det\left(\frac{\rho/N}{1 + \sigma_{\tilde{\mathbf{e}}}^2\rho} \mathbf{W}^{(l)}\right) \quad (8)$$

where the parameter u represents the size ($u \times u$) of each principal submatrix $\mathbf{W}^{(u)}$; $u = 1, \dots, L$ constructed from 1^{st} to u^{th} rows and columns of \mathbf{W} . Moreover, we have from [12] that $\mathbb{E}\{\sum_{i=1}^u \det(\mu \mathbf{A}^{(i)})\} = \mu^u \sum_{i=1}^u \mathbb{E}\{\det(\mathbf{A}^{(i)})\}$. Therefore, the mean of (8) can be found with $\mathbb{E}\{1\} = 1$ as

$$\begin{aligned} \mathbb{E}\{\det(\mathbf{\Delta})\} &= \mathbb{E}\{1\} + \sum_{u=1}^L \mathbb{E}\left\{\sum_{l=1}^u \det\left(\frac{\rho/N}{1 + \sigma_{\tilde{\mathbf{e}}}^2\rho} \mathbf{W}^{(l)}\right)\right\} \\ &= 1 + \sum_{u=1}^L \left(\frac{\rho/N}{1 + \sigma_{\tilde{\mathbf{e}}}^2\rho}\right)^u \sum_{l=1}^u \mathbb{E}\{\det(\mathbf{W}^{(l)})\} \end{aligned} \quad (9)$$

and from [12, Theorem III.2], the term $\sum_{l=1}^u \mathbb{E}\{\det(\mathbf{W}^{(l)})\}$ can be simplified using (5) and $D = M - M_s$ to

$$\begin{aligned} &\sum_{l=1}^u \sum_{i=1}^u \left[\det(\tilde{\mathbf{R}}_r^{(l)}) \det(\mathbf{R}_t^{(i)}) \mathbb{E}\{\det(\mathbf{G}_s^{(l)}) \det(\mathbf{G}_s^{H(i)})\} \right] \\ &= u! \left[\sum_{l=1}^u \det(\tilde{\mathbf{R}}_r^{(l)}) \right] \left[\sum_{i=1}^u \det(\mathbf{R}_t^{(i)}) \right] \\ &= u! \left[\left(1 + \frac{D}{u}\right) \binom{M_s}{u} \det(\tilde{\mathbf{R}}_r^{(u)}) \right] \left[\binom{N}{u} \det(\mathbf{R}_t^{(u)}) \right]. \end{aligned} \quad (10)$$

Using (9) and (10) in (7), a closed-form approximation of the ergodic capacity can be written as

$$\begin{aligned} \bar{C} \approx \log_2 \left[1 + \sum_{u=1}^L \left(\frac{\rho/N}{1 + \sigma_{\tilde{\mathbf{e}}}^2\rho} \right)^u u! \left(1 + \frac{D}{u} \right) \binom{M_s}{u} \binom{N}{u} \det(\tilde{\mathbf{R}}_r^{(u)}) \det(\mathbf{R}_t^{(u)}) \right]. \end{aligned} \quad (11)$$

For the special case of uncorrelated channel with perfect CSIR, $K = 1$, and $M = M_s$ (i.e. without RAS), we remark that (11) will be reduced to $\log_2 \left[1 + \sum_{u=1}^L \left(\frac{\rho}{N} \right)^u u! \binom{M}{u} \binom{N}{u} \right]$ which provide similar results to that of (29) in [12] under the same number of utilized antennas. Moreover, at high SNR, the L^{th} -order term in the logarithm of above capacity expression becomes dominant, and consequently we have

$$\bar{C} \approx L \log_2 \left[\left(\frac{\rho/N}{1 + \sigma_{\tilde{\mathbf{e}}}^2\rho} \right) L! \left(1 + \frac{D}{L} \right) \binom{M_s}{L} \binom{N}{L} \det(\tilde{\mathbf{R}}_r^{(L)}) \det(\mathbf{R}_t^{(L)}) \right]. \quad (12)$$

Note that the derivation of capacity equations are not affected when different RAS algorithms are considered. Consequently, optimal results can be achieved by employing capacity based selection methods [4]-[8], and suboptimal outcomes can be found for instance when norm based

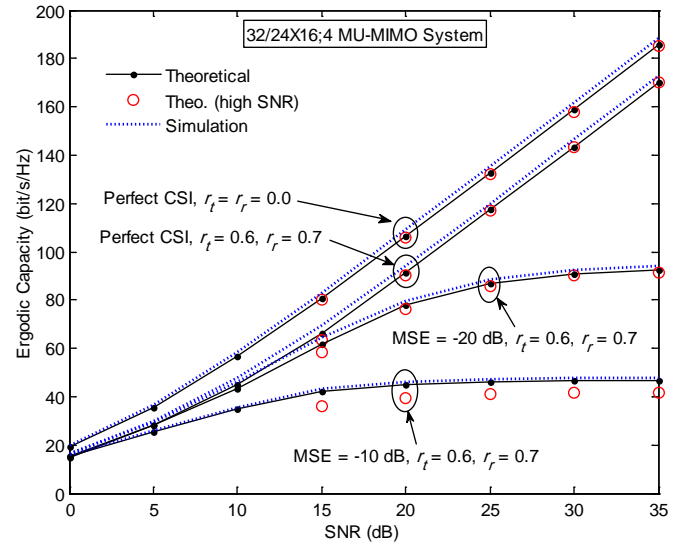


Figure 1. The capacity of 32/24 × 16; 4 system over correlated channel ($r_t = 0.6$ and $r_r = 0.7$) as a function of SNR for perfect and imperfect CSI (MSE = -10 dB, -20 dB). Results for uncorrelated channel ($r_t = r_r = 0$) and perfect CSI are also included for comparison.

TABLE I. SUMMARY OF THE CAPACITY RESULTS IN BIT/S/Hz FOR 32/24 × 16; 4 SYSTEM AT SNR OF 10 dB AND 30 dB WITH CORRELATION FACTORS AND ESTIMATION ERRORS CONSIDERED IN FIG.1

CSI Error:	Spatial Corr.		SNR = 10 dB		SNR = 30 dB	
	r_t	r_r	\bar{C} (theo.)	\bar{C} (sim.)	\bar{C} (theo.)	\bar{C} (sim.)
Perfect	0.0	0.0	56.9	58.5	159.1	161.8
Perfect	0.6	0.7	44.9	47.8	143.5	146.4
-20	0.6	0.7	43.2	46.3	90.8	92.5
-10	0.6	0.7	34.9	35.9	46.7	47.8

selection is utilized [6], [20]. Besides, for single user MIMO (i.e. point-to-point communications when $K = 1$), the capacity expressions can be used for transmit AS (instead of RAS) by considering all receive parameters ($M, M_s, \tilde{\mathbf{R}}_r^{(u)}$) for the transmitter and vice-versa.

IV. NUMERICAL RESULTS

In this section, numerical results of the ergodic capacity (11) and its approximation (12) are presented and verified with Monte Carlo averaging of (6) over 10^4 channel realizations. For notational convenience, the considered MU-MIMO system is represented as $(M/M_s \times N; K)$ where $N_T = N/K$. The optimal capacity based RAS algorithm is utilized to select the best M_s out of M antennas for all obtained results. To demonstrate the accuracy of our derived expressions, different LTE configurations towards next-generation massive MIMO are investigated over different channel conditions. The mean square error of CSI is shown in dB as $\text{MSE} = 10 \log_{10}(\sigma_{\tilde{\mathbf{e}}}^2)$ while the impact of spatial correlation at each transmitter ($\mathbf{R}_{t,k}$; $k = 1, \dots, K$) and BS receiver (\mathbf{R}_r) is carried out using the typical exponential correlation matrix model as [13], [18]

$$[\mathbf{R}_{t,k}]_{ij} = r_t^{|i-j|}; \quad i, j = 1, \dots, N_T; \quad r_t \in [0, 1] \quad (13)$$

$$[\mathbf{R}_r]_{ij} = r_r^{|i-j|}; \quad i, j = 1, \dots, M; \quad r_r \in [0, 1] \quad (14)$$

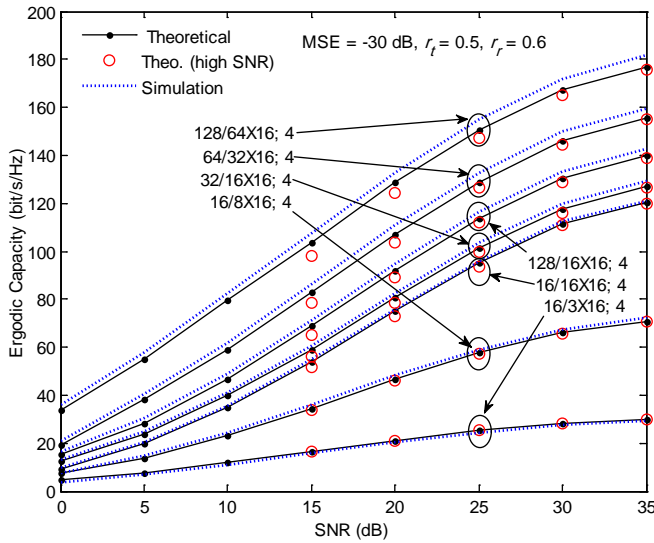


Figure 2. The capacity for different $M/M_s \times 16; 4$ configurations as a function of SNR for $r_t = 0.5$, $r_r = 0.6$, and $MSE = -30$ dB.

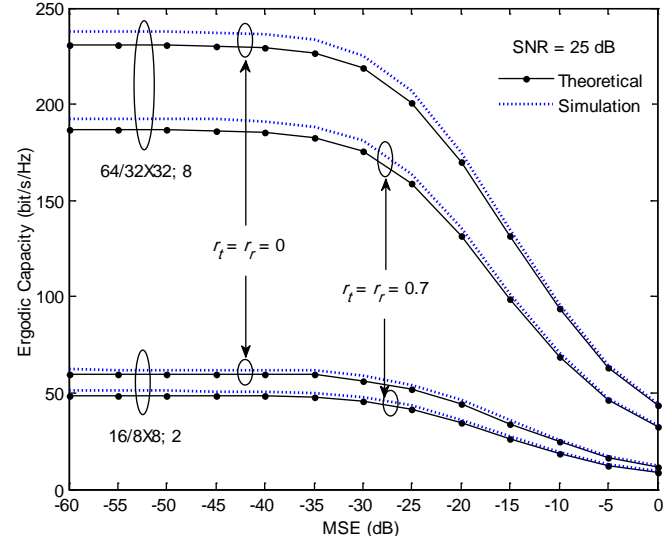


Figure 4. The capacity of $16/8 \times 8; 2$ and $64/32 \times 32; 8$ systems at SNR of 25 dB as a function of MSE, for $r_t = r_r = 0$ and $r_t = r_r = 0.7$.

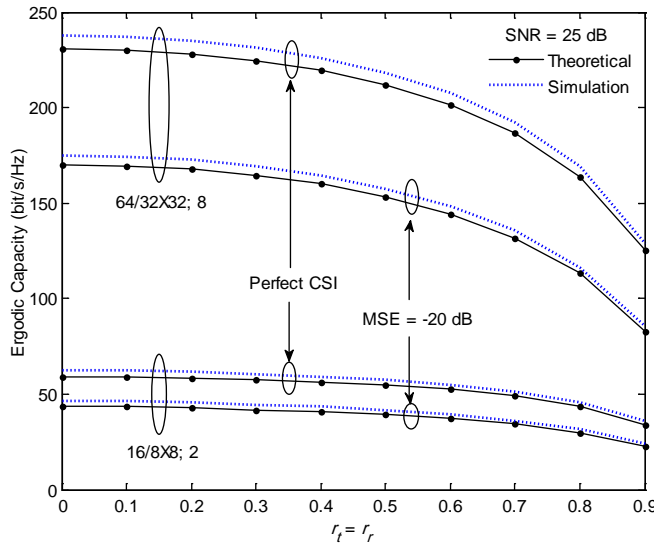


Figure 3. The capacity of $16/8 \times 8; 2$ and $64/32 \times 32; 8$ systems at SNR of 25 dB as a function of $r_t = r_r$, for perfect CSI and $MSE = -20$ dB.

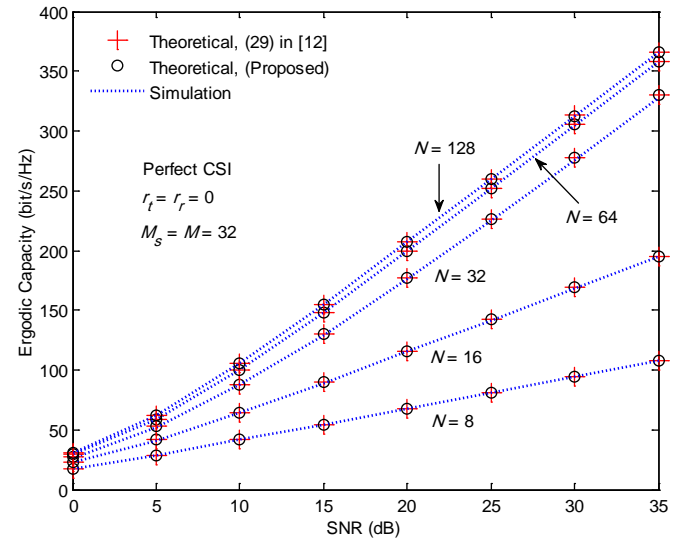


Figure 5. The capacity of $32/32 \times N; 1$ systems as a function of SNR for perfect CSI and $r_t = r_r = 0$ where $N_T = N$ and $M_s = M = 32$.

where r_t and r_r are correlations between any pair of adjacent antennas at each transmitter and the receiver, respectively.

In Fig. 1, capacity of $32/24 \times 16; 4$ system with $r_t = 0.6$ and $r_r = 0.7$ is shown as a function of SNR for perfect CSI ($\sigma_{\mathcal{E}}^2 = 0$) and MSE of -10 dB and -20 dB. Reference results for the uncorrelated channel ($r_t = r_r = 0$) with perfect CSI are also shown. It can be seen that the theoretical results are quite tight with the simulation outcomes for the entire range of SNRs. It is also shown as expected that the capacity decreases with the appearance of channel correlation and CSI error. Summary of the achieved results at SNR of 10 dB and 30 dB is presented in Table I. For example at SNR of 10 dB, the capacity difference is 1 bit/s/Hz for the correlated channel with $MSE = -10$ dB and 1.6 bit/s/Hz for the uncorrelated channel with perfect CSI. These values are slightly increased at high SNR of 30 dB to 1.1 and 2.7 bit/s/Hz, respectively.

Fig. 2 demonstrate the capacity of different $M/M_s \times 16; 4$ configurations as a function of SNR for $r_t = 0.5$, $r_r = 0.6$,

and $MSE = -30$ dB. Fixed ratio of $M/M_s = 2$ is implemented as $16/8$, $32/16$, $64/32$, and $128/64$ while increased ratio is considered as $16/16$, $16/3$, and $128/16$. As can be seen, without RAS, the analytic capacity of $16/16$ scenario is very tight with the simulation results for all SNRs where $D = 0$ in (11). With RAS, the difference between theoretical and simulation outcomes is slightly increased as the spectral efficiency increases considerably for larger M and/or M_s . For instance at SNR of 25 dB, the analytical capacity for $16/8$ and $128/64$ scenarios with fixed ratio is less than the simulation outcomes by 1.5 and 4.2 bit/s/Hz, respectively. Moreover, the closeness of the results is also demonstrated through different higher ratios. For example, the achieved differences between theoretical and simulation results for $16/3$ and $128/16$ at SNR of 25 dB are 0.6 and 2.5 bit/s/Hz, respectively. Notice that the capacity of $64/32$ and $128/64$ schemes with fixed ratios outperforms that of $128/16$ of higher ratio where for $N = 16$, the additional diversity gain

TABLE II. SUMMARY OF THE CAPACITY RESULTS IN BIT/S/Hz FOR $16/8 \times 8; 2$ AND $64/32 \times 32; 8$ SYSTEMS AT SNR = 25 dB ASSUMING $r_t = r_r = 0$ AND $r_t = r_r = 0.7$ FOR PERFECT AND IMPERFECT CSI OF MSE = -20 dB.

CSI Error:	Spatial Corr.		$16/8 \times 8; 2$		$64/32 \times 32; 8$	
MSE (dB)	r_t	r_r	\bar{C} (theo.)	\bar{C} (sim.)	\bar{C} (theo.)	\bar{C} (sim.)
Perfect	0.0	0.0	59.0	62.2	230.8	237.8
Perfect	0.7	0.7	49.4	51.8	187.5	193.7
-20	0.0	0.0	44.1	46.2	170.2	175.0
-20	0.7	0.7	34.4	36.5	131.9	136.5

from $M_s > 16$ RF chains has more impact on the capacity compared to that achieved from $M > 16$ antennas when connected with $M_s = 16$ RF chains.

On the other hand, the capacity approximations (11) and (12) are derived from the lower bound (6) with the expectation of concave function, and therefore, the theoretical curves lies below that of simulations for moderate to large values of L (see Figs. 1 and 2 for $L > 3$). Otherwise, the theoretical results will be above the simulations as shown in Fig. 2 for $16/3$ system. Furthermore, it can be seen from Figs. 1 and 2 that the capacity results of (12) at high SNR are quite tight to that of (11), mainly for $\text{SNR} \geq 20$ dB and $\text{MSE} \leq -20$ dB. For high level of MSE (e.g. -10 dB), the term $(\rho/N)/(1 + \sigma_{\epsilon}^2 \rho)$ in (12) becomes less than one, and hence the wider gap between the results as L increases.

In Fig. 3, the capacity of $16/8 \times 8; 2$ and $64/32 \times 32; 8$ systems at SNR of 25 dB is shown as a function of correlation parameters $0 \leq (r_t = r_r) \leq 0.9$ for perfect CSI and $\text{MSE} = -20$ dB. On the other hand, Fig. 4 presents the outcomes as a function of MSE for uncorrelated and correlated channels assuming $r_t = r_r = 0$ and $r_t = r_r = 0.7$, respectively. From these two figures, it can be seen as expected that the capacity decreases considerably as the correlation and/or MSE values increased toward high levels. In addition, the analytical and simulation results are tight to a certain extent for the entire ranges of correlations and MSE. Summary of the capacity outcomes of considered schemes is presented in Table II for $r_t = r_r = 0$ and $r_t = r_r = 0.7$ with perfect CSI and $\text{MSE} = -20$ dB. For $64/32 \times 32; 8$ configuration of high spectral efficiency, the difference between theoretical and simulation results is about 4.6 bit/s/Hz for the case of $r_t = r_r = 0.7$ and $\text{MSE} = -20$ dB, while 7 bit/s/Hz is shown for $r_t = r_r = 0$ and perfect CSI. Under same conditions, the difference is less for $16/8 \times 8; 2$ scheme of 2.1 and 3.2 bit/s/Hz, respectively.

In Fig. 5, the capacity results for specific case of $32/32 \times N; 1$ systems (without RAS) over uncorrelated channel with perfect CSI are shown for $N = 8, 16, 32, 64$ and 128. As can be seen clearly, the tightness of the derived expression (11) is demonstrated for all examined configurations compared with the simulation results and those found from (29) in [12].

V. CONCLUSION

As a key design objective for wireless MU-MIMO systems with RAS technique, the ergodic capacity has been assessed based on a new closed-form approximation over realistic channel environment of spatial correlation and imperfect CSI. The mathematical capacity expression has been derived in terms of channel MSE, utilized RF chains, and correlation matrices at transmitters and receiver. A simple analytical formula is presented also for the high SNR scenario. The

achieved results extend the state-of-the-art and provide more flexible approach for efficient evaluation of varied systems. Over different system configurations including future massive MIMO paradigms, the analytical outcomes are shown to be in a good agreement with the numerical simulations for the entire ranges of SNRs, MSEs, correlation levels, and arbitrary numbers of allowed users and utilized antennas and RF chains.

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